Damage Assessment Using Hyperchaotic Excitation and Nonlinear Prediction Error

S. TORKAMANI, E. A. BUTCHER, M. D. TODD and G. PARK

ABSTRACT

The idea of damage assessment based on using a steady-state chaotic excitation and state space embedding, proposed during the recent few years, has led to the development of a computationally feasible SHM technique based on comparisons between the geometry of a baseline attractor and a test attractor at some unknown state of health. This study explores an extension to this concept, namely a hyperchaotic excitation. The feature that is used to analyze the responses of the structures to the chaotic/hyperchaotic excitations is called 'Nonlinear Auto-Prediction Error' (NAPE), which is based on attractor geometry. A comparison between the results from the chaotic excitation with the results from each of the hyperchaotic excitations, obtained both numerically and experimentally, highlights the higher sensitivity of hyperchaotic excitations relative to a chaotic excitation.

INTRODUCTION

One aspect of damage assessment that has received less attention is the excitation. The reason is due to the nature of commonly-used features such as modal-based features. Since the modal properties are transient dynamic properties, the excitation is simply chosen to excite a target bandwidth (e.g., broad-band white noise). In comparison, Todd et al [1-3] used a method wherein a deterministic dynamic rather than the common stochastic white noise is applied to the structure for the sake of interrogation and the features based on steady-state dynamics are extracted to identify the damage. A steady-state chaotic excitation is shown by Todd *et al* [4] to satisfy the required conditions and the low-dimensional response to this type of excitation is proven to have the capability of being a sensitive indicator of damage. Thus the approach makes use of the intrinsic high sensitivity of chaotic systems to subtle changes of the parameters and has turned out to a computationally feasible SHM technique based on comparisons between the geometry of a baseline attractor and a test attractor at some unknown state of health. Features utilizing attractor-based techniques include auto-prediction error [5],

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cross-prediction error [4], local attractor variance [1,3], continuity[6], chaotic amplification of attractor distortion [7], and generalized interdependency [8].

In previous work of the current authors [9], a novel structural excitation, namely a hyperchaotic excitation, is employed and it is shown that through using a hyperchaotic attractor that possesses more than one positive Lyapunov exponent (PLE), the sensitivity of the technique can be enhanced due to stretching of the phase space in multiple directions. The attractor-based feature that is used in the previous work is 'average local attractor variance ration' (ALAVR) and the hyperchaotic attractor is shown to make this features more sensitive due to a more generous stretching of the phase space in multiple directions and thus allowing the trajectory to more fully explore the entire phase space. In this current study, *nonlinear auto-prediction error* (NAPE) is selected to serve as the damage-sensitive feature and an attractor-based predictive model of a healthy structure's dynamics is used to make predictions for the dynamics of a damaged structure and the severity of the damage is estimated by the accuracy of the prediction. The NAPE feature is applied with a hyperchaotic excitation to see whether or not this feature also confirms the higher sensitivity of a hyperchaotic excitation. Moreover, a hyperchaotic excitation having three positive Lyapunov exponents is also exploited here for the first time and the sensitivity of this excitation is compared with a two-positive-Lyapunov-exponent hyperchaotic excitation. Prior to empirically verifying the results using a portal frame test setup, numerical simulation using an 8-degree-of-freedom model is performed.

PHASE SPACE PROBLEM FORMULATION

Hyperchaotic Interrogation

Hyperchaos can be defined as chaotic behavior where at least two Lyapunov exponents are positive. Having all the advantages that make a chaotic signal suitable for being used as an excitation, it is hypothesized that a hyperchaotic signal should be even more sensitive to subtle changes in damage severity as a result of the trajectory being permitted to more fully explore the entire phase space. Thus, hyperchaotic interrogation can be an alternative excitation mechanism in damage detection when extra sensitivity to damage is required.

Tuned Excitation

Consider a simple linear N-degree-of-freedom structure forced with the output of a separate dynamical process governed by the function $\boldsymbol{F}$ as

$$\begin{gathered} \dot{\mathbf{x}} = \boldsymbol{F}(\mathbf{x}) \\ \dot{\mathbf{z}} = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{x}(t) \end{gathered} \tag{1}$$

where the chaotic/hyperchaotic signal $\mathbf{Bx}(t)$ is acted on by the filter described by the matrix $\mathbf{A}$. The vector $\mathbf{x}(t)$ is assumed to be the state-vector of a chaotic/hyperchaotic oscillator and the matrix $\mathbf{B}$ in this case simply serves to select a component of the oscillator to be filtered and the degree of freedom of the structure to be excited. For a linear system, the Lyapunov exponents are the real parts of the eigenvalues of $\mathbf{A}$ so the complete spectrum for the filtered chaotic/hyperchaotic signal $\boldsymbol{z}(t)$ is a combination of the exponents associated with the M-dimensional

chaotic/hyperchaotic system, λ_i^c, and the exponents of the N-dimensional filter, λ_j^L, where $i = 1, \ldots, M$ and $j = 1, \ldots, N$. The fractal dimension of the entire system is controlled by Kaplan-York conjecture. Damage to the system will result in changes to the eigenstructure of $\mathbf{A}$. This in turn will alter the structure's Lyapunov spectrum, which, for a linear system, consists of the real parts of the eigenvalues of $\mathbf{A}$. However, in order for this technique to have the best performance the excitation should be tuned for the structure. There are two tuning criteria based on attractor dimensionality. First, the Lyapunov spectrum of the oscillator must overlap that of the structure. This ensures that changes to the LE's of the structure, *i.e.*, by damage, will alter the dimension of the filtered signal. The degree of overlap, d_o, determines the extent to which the structure's dynamics are excited or, alternatively, the number of dimensions the structure is adding to the phase space. Second, the dominant exponent associated with the oscillator must be minimized for a given degree of overlap in order to maintain the lowest possible dimensionality.

Nonlinear Prediction Error as a Feature

Points on the 'damaged' attractors can be forecast trough applying a simple prediction scheme and by using a baseline attractor as a model. The main idea of all types of nonlinear prediction error features is that the presence of damage causes the baseline attractor to lose its prediction ability. The error defined as the difference between the predicted point and the measured point —prediction error— has been shown to be a good candidate to serve as a feature for both the detection and the assessment of structural damages. If the attractor in a baseline structural condition is compared to itself in a subsequent condition, the feature is called Nonlinear Auto-prediction Error (NAPE). But if the relationship between attractors obtained from multiple measurement points of the structure is studied in different structural conditions, the feature is called Nonlinear Cross-Prediction Error (NCPE). This study exploits the auto-prediction error as a feature (Figure 1).

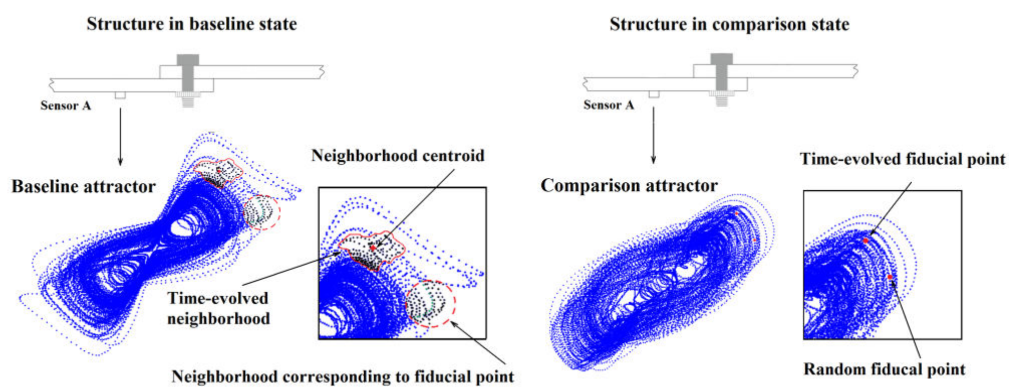

Figure 1. Nonlinear auto-prediction error feature.

The algorithm to calculate nonlinear auto-prediction error starts with randomly selecting a set of fiducial points on a reconstructed comparison attractor and finding the set of corresponding fiducial points on the baseline reconstructed attractor. Finally the prediction error for each fiducial point is formed through calculating the Euclidian distance between the predicted point for the comparison attractor and the time evolved correlated fiducial point. The detailed algorithm can be found in [10].

NUMERICAL SIMULATION

The structure used for numerical simulations is a linear n-degree-of-freedom spring–mass–damper structure where the first spring is connected to the ground and the excitation is just exerted to the last (n^{th}) mass of the chain. For $n = 8$ the equation of this system can be put into the form of Eq.(1) using the masses, spring stiffnesses and damping coefficients $m_i = 0.01,\ k_i = 2.0,\ c_i = 0.075$ where $i = 1, 2, \dots, 8$. Damage is introduced as a percentage of stiffness reduction of the first spring in the chain for this 8-DOF structure.

The main purpose of the current study is to compare the efficiency of chaotic and hyperchaotic oscillators when used as an excitation. Therefore some chaotic and hyperchaotic oscillators are used in this numerical simulation to compare their sensitivities to a small change in the structure's model. Here the most celebrated chaotic oscillator, Lorenz, as shown below is used.

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1)\delta \\ \dot{x}_2 &= (rx_1 - x_2 - x_1x_3)\delta \\ \dot{x}_3 &= (x_1x_2 - bx_3)\delta \end{aligned} \tag{2}$$

where $= 10$, $r = 28$, $b = 8/3$ and δ is a bandwidth control parameter which is used to tune the Lyapunov exponents of the excitation. Values less than unity for the parameter δ decrease the bandwidth of the input, while values greater than unity increase the bandwidth. The Lyapunov spectrum of the chaotic Lorenz system for the case of $\delta = 1$ has one positive and one zero LE's as $\lambda^C_{i=1,2,3} = 0.9056, 0.0000, -14.5723$. For the case of hyperchaotic excitation, we use the hyperchaotic version of the classic Lorenz system as the first trial. The hyperchaotic four-dimensional Lorenz [11] system has the form of

$$\begin{aligned} \dot{x}_1 &= (\sigma(x_2 - x_1) + x_4)\delta \\ \dot{x}_2 &= (rx_1 - x_2 - x_1x_3)\delta \\ \dot{x}_3 &= (x_1x_2 - bx_3)\delta \\ \dot{x}_4 &= (dx_4 - x_1x_3)\delta \end{aligned} \tag{3}$$

where the coefficients σ, r, b and d are selected as $\sigma = 10, r = 28, b = 8/3, d = 1.3$. The Lyapunov spectrum of the hyperchaotic Lorenz system for the base case of $\delta = 1$ is $\lambda^C_{i=1,2,3,4} = 0.39854, 0.24805, 0.0000, -12.913$ where there are two PLE's existing. This hyperchaotic system is called hyperchaotic Lorenz (2PLE's) from now on in this study. In order to have a better insight about a hyperchaotic system with more than two PLE's, the following version of the Lorenz oscillator is considered [12]:

$$\begin{aligned} \dot{x}_1 &= (\sigma(x_2 - x_1) + x_4)\delta \\ \dot{x}_2 &= (rx_1 - x_2 - x_1x_3 - x_5)\delta \\ \dot{x}_3 &= (x_1x_2 - bx_3)\delta \\ \dot{x}_4 &= (dx_4 - x_1x_3)\delta \\ \dot{x}_5 &= (kx_2)\delta \end{aligned} \tag{4}$$

Here, the coefficients σ, r, b are selected as $\sigma = 10, r = 28, b = 8/3$ and the two new parameters d, k as $\mathrm{d} = 2, \mathrm{k} = 10$. The Lyapunov spectrum of this oscillator for the case of $\delta = 1$ is $\lambda_{i=1,2,\dots,5} = 0.4580,\ 0.3371,\ 0.0415,\ 0.0000$ and -12.5046. As clear from the Lyapunov spectrum, this oscillator has three PLE's and it is called Lorenz hyperchaotic (3PLE's) in this study.

The Lyapunov spectrum of the linear 8-DOF system above, is equal to the real part of the eigenvalues with a negative sign, which is $\lambda_j^L = -0.127, -1.123, -2.980, -5.447, -8.192, -10.843, -13.042, -14.493$. As mentioned previously, two criteria must be met in this method. Firstly, the Lyapunov spectrum of the oscillator must overlap that of the structure. Secondly, the dimension of the oscillator after being filtered should be kept as low as possible. In order for these two criteria to hold, for each specific degree of overlap a variety of δ can be chosen. The admissible range of parameter δ based on the two tuning criteria for some degrees of overlap, along with the Kaplan-Yorke dimension of the excitation (input signal) and the structural response (filtered signal) for the chaotic Lorenz (Eq. (2)) and two hyperchaotic Lorenz oscillators (Eq.(3) and Eq. (4)) are listed in Table 1.

Table 1. Tuning and dimensionality of the chaotic and hyperchaotic excitations.

Chaotic/hyperchaotic Oscillators	Degree of overlap	Admissible δ	Dimension (D_L)	
			Excitation	Response
Chaotic Lorenz (1 PLE)	$d_o = 1$	$0.008 < \delta < 0.140$	2.06	$2.06 < D_L < 3$
	$d_o = 2$	$0.140 < \delta < 1.380$		$3 < D_L < 4$
Hyperchaotic Lorenz (2 PLE's)	$d_o = 1$	$0.01 < \delta < 0.196$	3.05	$3.05 < D_L < 4$
	$d_o = 2$	$0.196 < \delta < 1.933$		$4 < D_L < 5$
Hyperchaotic Lorenz (3 PLE's)	$d_o = 1$	$0.01 < \delta < 0.151$	4.07	$4.07 < D_L < 5$
	$d_o = 2$	$0.151 < \delta < 1.494$		$5 < D_L < 6$

With the excitation chosen to be each of the tuned Lorenz chaotic/hyperchaotic oscillators described above and the structure chosen to be the 8-DOF system, the equations of the structure and that of the oscillator were integrated simultaneously in the form of Eq. (1). An 8th-order Runge-Kutta algorithm with a fixed time step is employed for integration. The structure's response attractor is reconstructed using delay reconstruction wherein the reconstruction parameters, embedding dimension and embedding delay, are selected respectively based on 'False Nearest Neighbors' (FNN) method and 'Average Mutual Information' (AMI) function. As mentioned before, there is a wide range of parameter δ in each degree of freedom that the oscillator with any of these values of δ can be used as a tuned chaotic/hyperchaotic excitation. Therefore for the purpose of a sensitivity comparison between chaotic and hyperchaotic excitations using the 8-DOF structure, a parametric analysis is performed, in that the NAPE feature for 5% of damage to the structure is computed for a wide range of parameter δ covering the first two degrees of overlap. The values of NAPE obtained for 5 percent of damage at each δ is normalized by the value of NAPE obtained for undamaged case at the same δ used with the same excitation. Then the comparison is made among results obtained from each of three types of chaotic/hyperchaotic Lorenz oscillators as the excitation. The results of such comparison are illustrated in Figure 2.

The results show that the hyperchaotic interrogation technique exhibits a dramatically high sensitivity and the damage sensitive feature, NAPE, for some values of δ experiences a change of up to 400% for a damage level as small as 5%. For almost all values of δ except for a narrow range of $0.01 < \delta < 0.14$ at the beginning, the sensitivity of both hyperchaotic cases is by far higher than the chaotic case and the values of NAPE obtained from the hyperchaotic Lorenz (3PLE's) excitation is for most values of δ larger than that of hyperchaotic Lorenz (2PLE's) though not for all values of δ.

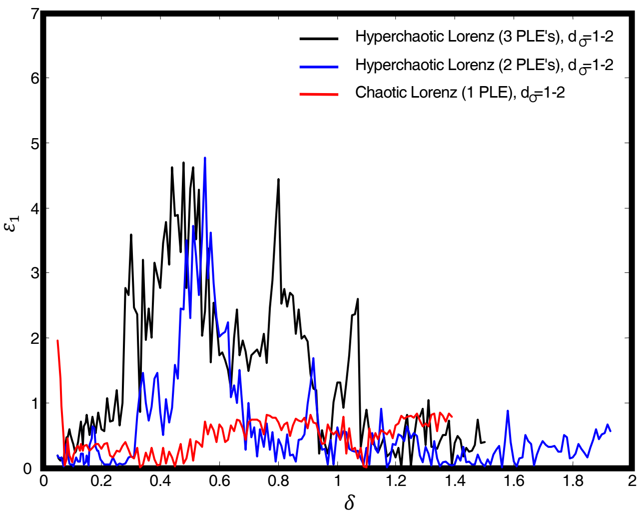

Figure 2. Normalized NAPE of chaotic and Hyperchaotic excitations for 5% of damage in the 8-DOF system for different values of δ within the range of first two degrees of overlap.

EXPERIMENTAL VALIDATION

The test setup used for experimental validation consists of a bolted-joint portal frame (shown in Figure 3) with two vertical and one horizontal element, connected via steel angle brackets. The vertical elements are also fixed to a base plate using angle brackets. As shown in the figure the base plate is clamped to the laboratory table to simulate a fixed boundary condition. The horizontal bolted joint of the bracket connecting the horizontal and vertical member on the farther side to the shaker is selected as the damaged joint to apply a joint preload loss damage scenario according to Table 2. A set of four accelerometers connected to the structure at different points depicted in Figure 3, are used for measuring the structural response.

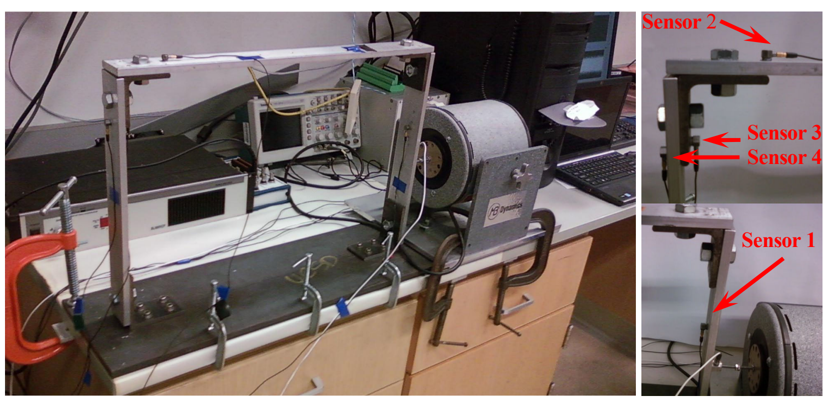

Figure 3.Portal frame test setup.

Table 2.list of the damage cases used for the portal frame experiment.

Damage case	**Condition**	**Preload**(Lb.in)
1 (baseline)	Fully Tight	more than 200
2	Tight	200 lb.in
3	Tight	100
4	Tight	50
5	Finger tight	less than 30
6	Loose with gap	----

An estimate of the largest Lyapunov exponent of the structure by assuming the structure to be linear and based on the measured modal properties is used for tuning the excitation. The excitation signals can be tuned to have a minimum degree of overlapping (based on the two tuning criteria) by knowing the largest Lyapunov exponent of the structure and choosing an appropriate value for the bandwidth control parameter δ. In order to obtain the chaotic/hyperchaotic excitation signals, a Matlab script is loaded to integrate the chaotic Lorenz of Eq.(2), the hyperchaotic Lorenz (2 PLE's) of Eq.(3) and the hyperchaotic Lorenz (3 PLE's) of Eq.(4) using bandwidth control parameters of $\delta = 600, 600$ and 500 respectively. The NAPE computed for the data measured from 4 channels (Figure 3) at each of the 6 damage levels (Table 2) is normalized using the NAPE obtained from baseline case of fully tight joint condition (case 1) from the same channel. Therefore the normalized NAPE obtained for case 1 turns out to be always zero. The results obtained from the chaotic Lorenz excitation (Eq. (2)) is compared with those of hyperchaotic excitations (Eq. (3) and (4)) in Figure 4.

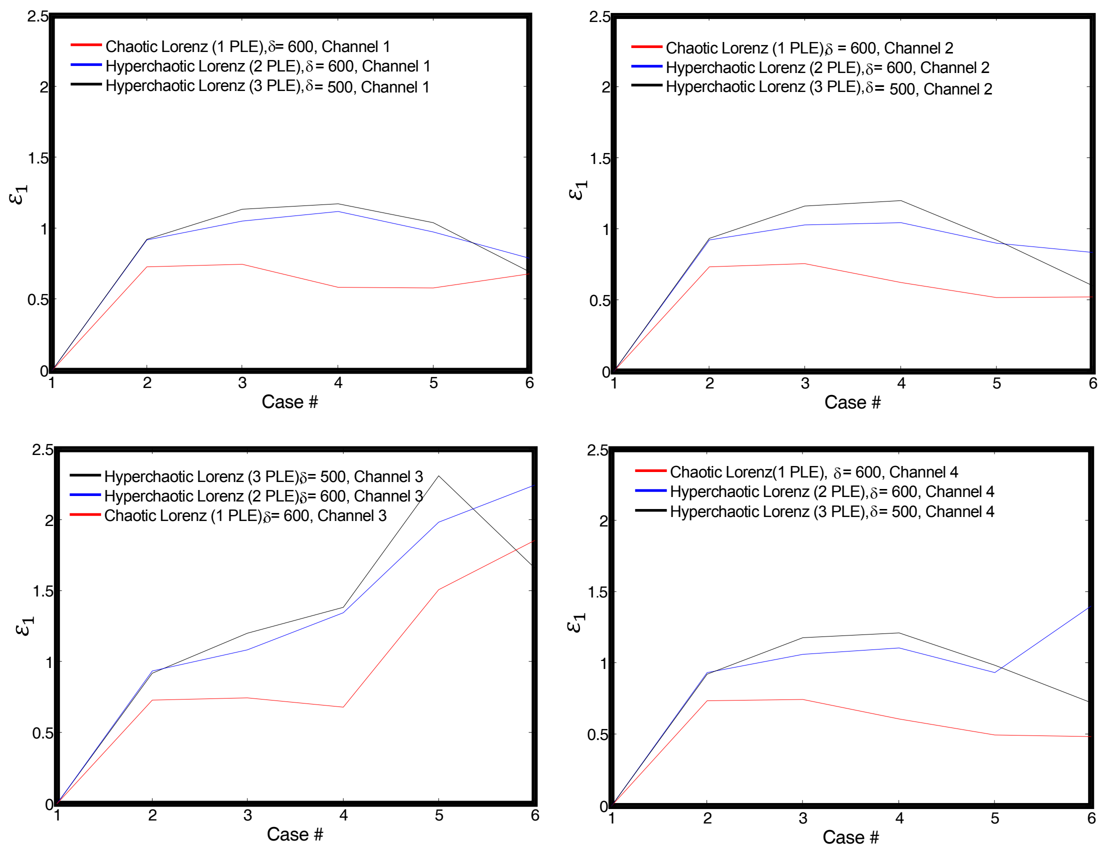

Figure 4. Comparison of the normalized NAPE versus damage case obtained from different channels when using chaotic and hyperchaotic Lorenz excitations.

As clear from Figure 4, the values of ε_1 for the data measured from channels 1, 2, and 4 for all types of excitations are almost flat for different excitations. For channel 3 measurements, ε_1 obtained from chaotic Lorenz starts exhibiting a significant rise not before the damage case 5 (finger tight) is applied. However, in cases of the hyperchaotic Lorenz excitations the ε_1 shows a rise (though a slight one) even with the damage case 3 (still with 100 lb.in preload). Both of the hyperchaotic Lorenz excitations show a bigger rise in damage case 5 as compared

to the chaotic Lorenz excitation. The hyperchaotic Lorenz (3PLE's) has the biggest rise among the other, though it is not exhibiting a considerable difference with that of hyperchaotic Lorenz (2PLE's). Hence, the results clearly highlight the higher sensitivity of the hyperchaotic Lorenz excitations compared with the chaotic Lorenz excitation.

CONCLUSIONS

This study extends the idea of chaotic interrogation technique by employing a hyperchaotic excitation, which is fundamentally new in its use as an excitation. A comparison between the numerical and experimental results from the chaotic excitation with similar results from hyperchaotic excitations (2PLE) when using NAPE feature, highlights the higher sensitivity of a hyperchaotic excitation relative to a chaotic excitation. Therefore, the NAPE feature also is shown to confirm the higher sensitivity of a hyperchaotic excitation. Moreover, a hyperchaotic excitation having three positive Lyapunov exponents is exploited here for the first time and it is shown that it can be in some cases even more sensitive than a two-positive-Lyapunov-exponent hyperchaotic excitation. It is demonstrated thus that it is not just the dimensionality of the excitation or the response attractor that controls the sensitivity of attractor-based features.

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